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Rotational dissipation and the Miesowicz coefficients

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In this work, we will study the relative contribution of each of the two dissipative channels of the Eriksen, Leslie, and Parodi (ELP) approach to the observed values of the Miesowicz viscosity coefficients of the nematic liquid crystals. According to the fundamental equation of the liquid crystal's viscosity dissipative process, $T\dot{S}=-\int d^3r\{\sigma_{ij}A_{ij}+\vec{h}\cdot\vec{N}\}$, there are two channels by which the nematic viscous dissipation can occur: or it occurs by means of a shear flow configuration, where A_{ij} is the characterizing term, or it occurs by means of a rotational configuration, where $\vec{N}$ is the characterizing term (these parameters will be defined in the paper). It will be also shown that this relative contribution can be measured by a simple relationship connecting the Miesowicz coefficients, which exhibits a quasitemperature independent behavior, suggesting that it is nearly constant through the entire domain of the nematic phase.

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I. INTRODUCTION

A simple idea to understand the origin of the nematic anisotropies is to attribute to the microscopic shape of their constituent molecules the source of their macroscopic anisotropies [1]. Accordingly, Hess and Baalss (HB) [1–5] assumed that if one could imagine a way by which the molecules of the nematic liquid crystals (NLC) could be continuously deformed, up to the point in which they become spherical, it would be possible to observe a corresponding reduction in the macroscopic anisotropies, until its vanishing. Inversely, if the idealized spherical molecules of an isotropic liquid could be deformed until they assumed the ellipsoidal form of an idealized molecule of nematic phase, the macroscopic physical properties would be transformed on those observed on the NLC. Essentially, these are the ideas of the (HB) conformal transformation hypotheses, originally applied to the study of the viscosity of an perfect ordered nematic fluid [1] and afterwards generalized to describe uniaxial phases with imperfect order [5]. Of course, such molecular deformations cannot be easily performed in the laboratory. But, mathematically, the spherical symmetric interaction of an isotropic fluid can be transformed in the ellipsoidal interaction of the nematic molecules (and vice versa) and the macroscopic consequences of these virtual transformations can be compared with the experimental data of the NLC. Using this approach Hess and co-workers, demonstrated that the anisotropic viscosity of a nematic liquid can be written in terms of the viscosity of an isotropic liquid. Notwithstanding the excellent explanation for the nematic phase produced by this idea [5], the final results need further improvements to explain the full range of the experimental data [7–9]. As we will show ahead, the comparison of the outcomes of the HB theory with the experimental data indicates that some fundamental aspects of the viscosity phenomena is yet missing in the HB approach.

The aim of this work it to fulfill this gap by studying the contribution of the rotational dissipation in the HB approach. In liquid crystals, the dissipation occurs by means of two processes: the first is determined by the shear fluid flow configuration, which is shared by all fluids, the another happens when the anisotropic molecules of the liquid crystal (LC) rotates with relation to the nematic fluid that circumscribe it. Usually, the HB approach is applied to fluid flow configurations that do not consider this last kind of dissipation; the nematic grains are taken as pointlike structures that do not rotate [1–6]. It will be shown here that when the rotational dissipation is considered, the HB approach becomes improved, and efficiently describes the nematic viscosity. The aim of this work is to obtain a procedure that can take care of the contribution of the director rotation to the HB description of the nematic viscosity and measure how this channel of dissipation is important to understand nematic rheological problems.

II. FUNDAMENTALS

When applied to NLCs, the dissipative source[12–18],

$$T\dot{S}=-\frac{d}{dt}\int d^3r\left\{\frac{1}{2}\rho v^2+F\right\} \tag{1}$$

(where $\rho v^2/2$ is the density of kinetic energy and F gives all kinds of configuration energies as defined, for example, by de Gennes [12]), shows that for these materials there are only two possible channels of energy dissipation, given by the formula,

$$T\dot{S}=-\int d^3r\{\sigma_{ij}A_{ij}+\vec{h}\cdot\vec{N}\}. \tag{2}$$

The first, characterized by a fluid flow with the form $A_{ij}=(\partial_i v_j+\partial_j v_i)/2$, describes the shear flow dissipation. The second, characterized by the term $\vec{N}=\{\dot{\vec{n}}-(\vec{w}\times\vec{n})]\}$, describes the rotation of the director with respect to the background fluid, and gives to the rotational dissipation. In these equations, $\vec{n}$ is the nematic director field, and the variables $\vec{v}$ and $\vec{w}$ are point dependent fluxes, where $\vec{v}$ is velocity of the fluid and $\vec{w}$ is the director angular velocity. Associated with each of these fluxes are the thermodynamical forces σ_{ij} and

h_i. The first is the dissipative stress tensor of the ELP approach,

$$\sigma_{ij} = \alpha_1 n_i n_j n_k n_l A_{kl} + \alpha_2 n_i \Psi_{jk} n_k + \alpha_3 n_j \Psi_{ik} n_k + \alpha_4 A_{ij} + \alpha_5 n_i A_{jk} n_k + \alpha_6 n_j A_{ik} n_k, \tag{3}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_6$ are the Leslie coefficients, $\Psi_{ik} n_k = \dot{n}_i - W_{ik} n_k$ describes the rotation of the director $\vec{n}$ relative to the background fluid, and $W_{ij} = (\partial_i v_j - \partial_j v_i)/2$. The second, $\vec{h}$, is the molecular field [12], which is proportional to the antisymmetric part, σ^A_{ij}, of the above stress tensor, $\sigma^A_{ij} = (h_i n_j - n_i h_j)/2$, and it is given by,

$$h_i = (\alpha_2 + \alpha_3) n_k A_{ik} + (\alpha_3 - \alpha_2) N_i. \tag{4}$$

Notice that A_{ij} and W_{ij} are, respectively, the symmetric and antisymmetric parts of the gradient of the fluid velocity

$$\partial_i v_j = A_{ij} + W_{ij}. \tag{5}$$

The aim of any microscopic pseudomolecular LCs rheological theory would be the determination of the values of the Leslie coefficients in terms of the temperature and the microscopic parameters of the theory. Nevertheless, notwithstanding the enormous amount of experimental data [19–23] (for a large set of different LCs materials) revealing the values assumed by these coefficients, a satisfactory theory for them does not exist [7,8]. Two approaches to this problem are found on the LCs literature. One of then is the Doi theory [24–28], which is the most widespread approach prescribing a microscopic model to the viscosity of the NLC. Nevertheless, even having the great merit of producing an expression free of adjustable parameters [29], capturing the essence of the phenomena and giving a semimicroscopic explanation to the origin of their anisotropy, it also presents well documented disagreements with the experimental data [7,8]. The other theory is the HB approach [1–5], which we will describe below.

Surely, the most secure affirmation about the values assumed by the Leslie coefficients follows from the Onsager microscopic time reversal relation [30], which was applied by Parodi [14] in Eqs. (3) and (4) above. It revealed that the Leslie coefficients are not independent, but connected: $\alpha_6 - \alpha_5 = \alpha_3 + \alpha_2$. So, when the Leslie approach is considered, only five independent combinations of the viscosity coefficients need to be effectively measured. Here, due to its straightforward physical interpretation and facility in which the corresponding experimental data can be found in the liquid crystal literature, we will use the combination of these coefficients that gives the Miesowicz coefficients, as defined, for example, by de Jeu [31]. To construct a model for the nematic stress tensor, a precise definition of the geometry in which each of them can be measured is needed. Suppose a NLC sample submitted to a fluid flows along the $\vec{e}_y$ direction, with the velocity varying along the $\vec{e}_z$ direction. Under these conditions, the relevant components of the stress tensor are those that, containing the derivative $\partial_z v_y$, are determined by the component σ_{zy}. A straightforward calculation shows that, when the director is fixed along each one of the directions $\vec{n} = (0,0,1)$, $\vec{n} = (0,1,0)$, and $\vec{n} = (1,0,0)$, the respective following set of the Miesowicz coefficients is obtained: $m_1 = \frac{1}{2}(\alpha_4 + \alpha_5 - \alpha_2)$, $m_2 = \frac{1}{2}(\alpha_3 + \alpha_6 + \alpha_4)$, and $m_3 = \frac{1}{2}\alpha_4$. Nevertheless, these are only three of all five independent viscosity coefficient predicted by the ELP approach. To obtain the remaining two viscosity coefficients, we can consider that when the director is not parallel to one of the three mutually orthogonal axes corresponding to m_1, m_2, and m_3, the observed viscosity would not be determined by a simple combination of the Miesowicz coefficients. Through an straightforward calculation, it can be shown that, if, for example, the director equally mixtures the directions corresponding to m_1 and m_2, $\vec{n} = \sqrt{2}(0,1,1)/2$, the corresponding viscosity, $m_{45°}$, is not given by the average of m_1 and m_2, but a new coefficient m_{12} arises, $m_{45°} = \frac{1}{2}(m_1 + m_2) + \frac{1}{4} m_{12}$, where $m_{12} = \alpha_1$. Finally, the last viscosity coefficient will be given by $\gamma_1 = \alpha_3 - \alpha_2$, which couples the antisymmetric stress tensor with the rotational of the velocity in Eq. (5), describing the viscosity associated with the rotation of a nematic grain [12]. In this work, we will be focused on the three Miesowicz coefficients.

A. HB approach

Before beginning the exposition of the fundamentals of the HB approach, let us make some appointments about the terminology that we will use to designate the nematic molecules. As the microscopic entities composing a nematic compound can be molecules or micelles [12], depending if the nematic material is thermotropic or lyotropic, and, as the HB approach can equally applied to both king of nematic liquids [8,9,32–36], the nematic grains will be conceived as ellipsoidal microscopic bodies, which synthesizes the geometrical shape of the microscopic nematic entities, and which can be obtained with the deformation of a spherical microscopic body. Now on, in the context of this work, we will interchange the use of the terms nematic grains, nematic molecules, and nematic micelles.

According to the HB hypotheses [1–5], the expression given at Eq. (3) would be obtained from the corresponding isotropic one through a affine transformation, $HB(\sigma_{ij})$. Namely, if $\sigma_{ij} = \eta \partial_i v_j$ gives the expression for the viscosity of an isotropic Newtonian fluid, then

$$\sigma^{HB}_{ij} = \eta \frac{ds^l}{dx^i} \frac{dx^j}{ds^k} (\partial_l v^k) \tag{6}$$

would correspond to the Leslie expression to the viscosity of a nematic liquid, where the letters x and s stand for ellipsoidal and spherical coordinates, respectively, as defined in Eq. (8) below. Consequently, by definition of affine transformation, the HB approach is composed by two different mathematical entities: (a) the linear transformation between the tensor components (ds^l/dx^i), and (b) the tensor components in which it acts, $(\partial_l v^k)$.

So, we can consider separately the role of these two mathematical quantities. The results of this paper are concerned with the part (b) of the points enounced above. Namely, the coefficients of ds^l/dx^i will not appear in our final results. Anyway, we will discuss ds^l/dx^i to give a precise form to the preceding statement.

In mathematical terms, the HB transformation can be obtained [7–9,32–35] by assuming that the ellipsoidal interaction potential Φ_E between molecules in NLC can be transformed in a spherical interaction potential Φ_S, once the vector $\vec{r}$ connecting two particles is submitted to an appropriated affine transformation. That is,

$$\Phi_E(r_S) = \Phi_S(r_E), \tag{7}$$

where the indexes E and S stand for ellipsoidal and spherical symmetries, respectively. Namely, the physics of the nonspherical potential, Φ_E, can be obtained from a spherical one Φ_S, if the distance between two points is given by a metric in which two points on the nonspherical equipotential surface become "equidistant" from a potential center. As a consequence, there would be an affine transformation leading the physics of liquids formed with spherically interacting molecules to the physics of liquids with ellipsoidal ones, and vice versa. Concretely, it is assumed that if s^i are the coordinates of an spherical equipotential surface S of an idealized molecule of an isotropy liquid, and x^i are the corresponding coordinates of a ellipsoidal surface E, obtained from S by a continuous deformation, then

$$ds^i = \frac{ds^i}{dx^j}dx^j, \quad \frac{d}{ds^i} = \frac{dx^j}{ds^i}\frac{d}{dx^j}, \tag{8}$$

which are connected by

$$\frac{ds^i}{dx^m}\frac{dx^m}{ds^j} = \delta^i_j, \tag{9}$$

relates distances measurements in these two surfaces.

The condition given at Eq. (7), which enforces that the strength of the molecular interaction along corresponding points of the surfaces S and E remains the same, can be obtained with the requirement that, for the spherically symmetric potential, the interaction is exclusively dependent on the distance. Consequently, the requirement given at Eq. (7) can be obtained with the imposition that the effective distance between points in the spherical and in the deformed system remains the same, that is,

$$ds^2 = \delta_{ij}ds^i ds^j = \delta_{ij}\frac{ds^i}{dx^k}\frac{ds^j}{dx^l}dx^k dx^l = g_{kl}\, dx^k dx^l, \tag{10}$$

where

$$g_{ij} = \delta_{lm}\frac{ds^l}{dx^i}\frac{ds^m}{dx^j}.$$

That is, the deformation of an isotropic system to an anisotropic one can be performed introducing a metric g_{ij}. In a previous work [9], it has been shown that the dependence of the physical quantities of the nematic phase on the order parameter S can be put on the parameters defining the metric g_{ij}. That is, the passage from an isotropic phase to an anisotropic one can be understood as the arising of a new metric, which, in consequence, must be temperature dependent.

B. To HB transformations for the shearing flow

We will give here an example that shows that the usual HB approach is not able to describe the experimental data of the nematic viscosity. Its importance stays in the fact that their conclusions do not depend on the parameters of the HB transformation, Eq. (11) below, indicating that the inadequacy of the HB approach must be related with the tensor in with the HB transformation acts. Ahead, we will retake this same example to show that the introduction of the rotational dissipation in the HB approach enhances its applicability and solves the problem present by this example. Let us assume that $\sigma_{ij} = \eta\partial_i v_j$ describes the stress tensor of an hypothetic isotropic liquid, with spherical molecules, then Eq. (6) would describe the stress tensor of a liquid composed by molecules interacting through a deformed potential whose deformation is given by dx^j/ds^k and ds^l/dx^i.

According to the reasoning followed at the previously quoted paper [9], a model of such deformations would be given by

$$\frac{dx^i}{ds^j} = r_b\delta_{ij} + (r_a - r_b)n_i n_j \quad\text{and}\quad \frac{ds_i}{dx_j} = \frac{1}{r_b}\delta_{ij} + \left(\frac{1}{r_a} - \frac{1}{r_b}\right)n_i n_j, \tag{11}$$

where $\{r_a, r_b\}$ are parameters that depend on the temperature of the nematic sample [9]. Geometrically, they can be interpreted as the as the effective dimensions of a uniaxial ellipsoid representing the thermalized shape of a calamitic nematic grain, r_a gives the long axis of the grain and r_b the symmetric minor axis.

A direct substitution of Eq. (11) in Eq. (6), and a comparison with Eq. (3) reveals that the three Miesowicz coefficients are given by

$$m_1 = \frac{r_a}{r_b}\eta$$

$$m_2 = \frac{r_b}{r_a}\eta,$$

$$m_3 = \eta. \tag{12}$$

These relations reveal that the conformal transformation hypotheses is able to predict an anisotropic behavior for the nematic viscosity [1–4]: for calamitic samples, the Miesowicz coefficients appear in the order observed experimentally $m_1 > m_3 > m_2$. Likewise, the experimental data also shows that in fact m_3 would behave as the extension in the nematic phase of the isotropic viscosity coefficient. Furthermore, we also see that these equations predict a universal relation between the Miesowicz coefficients. If we define a parameter r as

$$r = \frac{m_3}{m_1}\frac{m_3}{m_2}, \tag{13}$$

the use of Eq. (12) leads to

$$r = 1. \tag{14}$$

Showing that the above ratio between the Miesowicz coefficients would be completely independent of the parameters of the HB transformation. If true, this relation would furnish a strong support to the HB conformal transformation.

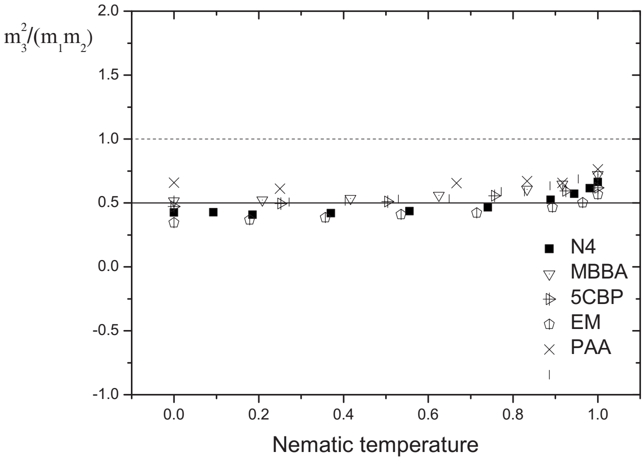

FIG. 1. Parameter $r = m_3^2/(m_1m_2)$ versus the uniformized nematic temperature scale (defined in the text). The straightforward application of the HB model predicts that $r=1$ (dashed line). Nevertheless, the experimental gives $r \approx \frac{1}{2}$ (continuous line). In this paper, we show that when we consider that the nematic grain is an extended particle, in which the rotational motion can lead to a dissipation process, with the same intensity of the shear flow channel [$\eta_1 = \eta_2$ in Eq. (22)], the result observed is this figure is found.

C. Experimental data

In order to verify whether the experimental results are in accordance with the relation deduced above, we have collected experimental data from the literature[19–23], and, in order to have a broad scope of such relations, only those data encompassing the entire range of the nematic phase have been considered. For the compounds listed in Fig. 1, a uniformized temperature scale was constructed [7–11] in such a way that for the nematic-crystalline (NC) the temperature $T=0$ was attributed while for the nematic-isotropic (NI) the temperature $T=1$ was attributed. That is, if T_{NI} is the Kelvin temperature of the NI and T_{NC} is the Kelvin temperature of the NC, then the nematic temperature t is related with the Kelvin temperature T by the relation

$$t = \frac{T - T_{NC}}{T_{NI} - T_{NC}}. \tag{15}$$

As it is shown in Fig. 1, the experimental data are in flagrant disagreement with Eq. (14), if a relation between the Miesowicz coefficients exists, it is not given by Eq. (14). At this work, it will be shown that a relationship between the viscosity coefficients in fact exists, but the above model is unable to find it.

III. NONSPHERICAL NEMATIC PARTICLE

The fact that the HB conformal transformation approach, either in the form that has been used here, or in the original form used by HB [1–4], does not reproduce the viscosity experimental data, is not enough to assure that it is not useful for the description the nematic viscosity. Equation (6) is in notorious contradiction with the fundamentals of the Hess approach, with the nematic rheological dissipation theory and with experimental facts. The essence of the conformal transformation stays in the deformation of an spherical equipotential surface into an elliptical one. Nevertheless, even furnishing an anisotropic result, the final equation does not full describe the degrees of freedom of an anisotropic particle: their rotational motion is not described by the velocity $\vec{v}$ that we have used. In Eq. (6), $\vec{v}$ describes the velocity of a pointlike particle and the HB transformation must be applied to extended spherical particles that are deformed to ellipsoidal ones. Namely, for an observer at the laboratory, the velocity $\vec{v}$ of an extended anisotropic rigid body is composed of two terms, one considering the motion of the its center of mass $\vec{v}^c$, and another considering the rotation of its internal points $\vec{\rho}$ around the center of mass. That is,

$$\vec{v} = \vec{v}^c + \vec{w} \times \vec{\rho}, \tag{16}$$

where $\vec{w}$ is the instantaneous angular velocity. Furthermore, according to the ELP approach [13–18] sketched above, the complete set of dissipative fluxes configurations present in the nematic dissipation expression are given by A_{ij} and $N_i = \dot{n} - (w \times n)$. Nevertheless, Eq. (6) does not takes care of N_i; only the dissipation associated with the shear flow A_{ij} is present in that expression. A term characterizing the rotational dissipation is missing in the above equation.

Furthermore, in order to make Eq. (16) consistent with the fact that a spherical grain does not have present rotational dissipation, we will consider $\vec{\rho}$ as a generalized coordinate, which extends without upper limit from the point $\vec{x}$, having the form $\vec{\rho} = \vec{r} - \vec{r}_o$, where $\vec{r}_o$ is the original ratio of the spherical equipotential surface that is deformed into a elliptical one, and, in order to guarantee that this terms disappears for a spherical symmetry body, we assume that $|\vec{r}| > |\vec{r}_o|$. With these definitions, we have the guarantee that $\vec{\rho}$ characterizes the anisotropy of the be grain. Therefore, it will be assumed that the HB transformation would not act on the stress tensor, $\sigma_{ij} = \eta \partial_i v_j$, that has been postulated at Eq. (6), but it would act on the extended form,

$$\sigma'_{ij} = \eta \{ \partial_l v_k^c + \partial_l [(\vec{w} \times \vec{\rho})_k] \}, \tag{17}$$

suggesting that the transformed equation would have the form,

$$\sigma_{ij}^{HB} = \eta \frac{ds^l}{dx^i} \frac{dx^j}{ds^k} \{ \partial_l v_k^c + \partial_l [(\vec{w} \times \vec{\rho})_k] \}, \tag{18}$$

which, as we have anticipated, does not changes the HB transformation, but the tensor on which it acts. Furthermore, as it stands, this equation cannot be used to compute viscosity coefficients, it depends on microscopic variables, such as $\vec{v}^c$ and $\vec{w}$, which must be averaged and expressed in terms of nematic variables before we can use it to further calculations. This has been done in the Appendix, where it has been found that once averaged, σ'_{ij} assumes the form,

$$\langle \sigma'_{ij} \rangle = \eta \{ A_{ij}(\vec{v}^c) + \bar{\rho} [\dot{n}_i - W_{ik}(\vec{v}^c) n_k] n_j \}, \tag{19}$$

where $\bar{\rho}$ is the mean value of radial variable ρ (defined above) at the border of the longest axis of the grain. The gratifying aspect of this result is that the rotational dissipative term that we have found is proportional to the expression of the rotation of the director with respect to the background, given at Eq. (2). In fact, it is difficult to imagine another form to it.

IV. ROTATIONAL VISCOSITY AND THE MIESOWICZ COEFFICIENTS

Here, we will accomplish the aim of this work, the HB transformation will be applied to Eq. (4). As η and $\eta\bar{\rho}$ appears as parameter fixing the relative contribution of the shearing and rotational dissipative fluxes of the nematic viscosity, without loss of generality we can assume that

$$\sigma_{ij}^{HB} = \frac{ds^l}{dx^i}\frac{dx^j}{ds^k}\{\eta_1 A_{ij}(\vec{v}^c) + \eta_2[\dot{n}_i - W_{ik}(\vec{v}^c)n_k]n_j\}. \quad (20)$$

That is, as the ELP approach has two dissipative channels, each of these coefficients represents one of then; η_1 will be the viscosity term already present in the usual HB approach and η_2 will be here introduced to attend the rotational viscosity. With them, we will express the relative contribution of that dissipative channels, the shear follow, and rotational dissipations. Using the definition of the Miesowicz coefficients, it is straightforward to show that

$$m_1 = \frac{r_a(\eta_1 + \eta_2)}{2r_b},$$

$$m_2 = \frac{r_b\eta_1}{2r_a},$$

$$m_3 = \frac{\eta_1}{2}, \quad (21)$$

which gives to the parameter r, defined at Eq. (14), the form

$$r = \frac{\eta_1}{\eta_1 + \eta_2}. \quad (22)$$

This equation shows that the nonexperimentally verified result found at Eq. (14) can be understood as long as the parameter fixing the rotational dissipation, η_2, is non-null. Furthermore, it also gives a meaning to the parameter r and Fig. 1; beyond the explicit ratio between the Miesowicz coefficients, this parameter also measures the relative contribution of the two main dissipative channels of Eq. (2). In fact, it depends only on it. Namely, the ELP approach has revealed that there are two channels by which the dissipative rheological process can occur in nematic materials; the shearing flow and the rotational flow. Suppose now that we want know how much dissipation occurs in each one of this channels. First, the set of Eqs. (21) reveals that only the Miesowicz coefficient m_1 has dependence on both channels of dissipation; m_2 and m_3 are uniquely determined by the shear follow dissipation term. Furthermore, the combination of these parameters, given by the parameter r, reveals an expression in which the dependence on the thermalized geometrical parameters, r_a and r_b, has been eliminated and only the relative dependence on the values of these dissipative coefficients has been preserved. Nevertheless, even losing it dependence of the parameters r_a and r_b, this relation presents a remarkable stability and independence of the temperature, indicating that the proportional composition of shear flow and rotational follow for the dissipative process may independent of the temperature. Figure 1 also shows that r assumes a nearly constant value $r \approx \frac{1}{2}$, along the entire range of the nematic phase (only at the neighborhoods of the NI phase transition is some deviation of this rule seems to be observed). Consequently, while m_2 and m_3 do not present rotational dissipation, we see that for m_1, we would have $\eta_1/\eta_2 \approx 1$, meaning that the contribution of the two dissipative channels would be approximately equal.

Beyond the theoretical important of these results, there is practical importance on them, the regularity in which the experimental data of Fig. 1 are distributed suggests a law that can be used to predict the value of a Miesowicz coefficient, once two of then are known.

V. CONCLUSION

With the use of the HB theory, we have calculated the contribution of the two dissipative channels of the ELP approach to the Miesowicz coefficients of NLCs. We have found that for the computation of m_1, we cannot avoid the contribution of the rotational viscosity terms. The procedure that assumes that the fixing of the director configuration would cancel the contribution of the rotational dissipation channel is clearly a fallacious reasoning; when the director is frozen, it only becomes fixed with relation to an observer in the laboratory, with relation to it surroundings the relative rotation remain existing.

Furthermore, we have found a meaning for the ratio $r = m_3^2/(m_1m_2)$ between the Miesowicz coefficients: without the presence of any other parameter, it expresses the relative contribution of the two dissipative channels of the ELP dissipation source to the Miesowicz coefficients.

Finally, the experimental data corresponding to this relationship has revealed a surprising regularity. They are distributed along a regular horizontal line, indicating that they have a very weak dependence on the temperature, or even does not depend on it. Such relationship can be used to predict the value of a given Miesowicz coefficient once the other two are known; the microscopic explanation of this regularity is not yet known.

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APPENDIX

In this , we will shown how the microscopic equation, given at Eq. (17), can be written in terms of the macroscopic nematic parameters, Eq. (19). Let us begin by making some initial simplifying suppositions. We will consider that $\vec{v}^c$ and $\vec{w}$ are fields: it will be assumed that at every point $\vec{x}$ of the sample, we can find a particle with angular velocity $\vec{w}$, and around it (or dragging it) a background fluid flow with velocity $\vec{v}^c$. Furthermore, in order to simplify our reasoning, we will assume that the nematic grain at the point $\vec{x}$ is rigid, $\partial_i \vec{w}=0$, for all i. With this supposition, let us consider the k component of the rotational of Eq. (16)

$$[\vec{\nabla}\times(\vec{v}-\vec{v}^c)]_k=\epsilon_{kji}\epsilon_{iqp}\partial_j(w_q\rho_p), \tag{A1}$$

which give us an expression for $\vec{w}$ in terms of $\vec{u}=\vec{v}-\vec{v}^c$

$$\vec{w}=\frac{1}{2}\vec{\nabla}\times\vec{u}. \tag{A2}$$

This equation allow us to write $\vec{w}\times\vec{\rho}$ as

$$(\vec{w}\times\vec{\rho})_i=\frac{1}{2}(\vec{\nabla}\times\vec{u})\times\vec{\rho}=-W_{ip}(\vec{u})\rho_p, \tag{A3}$$

where we have used $W_{ij}(\vec{u})=(\partial_i u_j-\partial_j u_i)/2$, the antisymmetric part of $\partial_j u_i$. By putting this equation in Eq. (16), it is found that,

$$\partial_j v_i=\partial_j v_i^c-\partial_j[\ W_{ip}(\vec{u})\rho_p)], \tag{A4}$$

which is the same as

$$\begin{aligned}\partial_j v_i&=A_{ij}(\vec{v}^c)+W_{ji}(\vec{v}^c)-\partial_j[W_{ip}(\vec{u})\rho_p]\\&=A_{ij}(\vec{v}^c)-\partial_j[W_{ip}(\vec{v})\rho_p]+[\partial_j W_{ip}(\vec{v}^c)]\rho_p.\end{aligned}$$

Now, inspired by Eq. (A3), we introduce an angular velocity $\vec{\phi}$, such that,

$$(\vec{\phi}\times\vec{\rho})_i=-W_{ip}(\vec{v})\rho_p, \tag{A5}$$

which, analogously with Eq. (A3) and Eq. (A2), can be written as

$$\vec{\phi}=\frac{1}{2}\vec{\nabla}\times\vec{v}=\frac{1}{2}(\vec{\nabla}\times\vec{v}^c+\vec{\nabla}\times\vec{u}). \tag{A6}$$

With this definition, it is found that,

$$\partial_j[W_{ip}(\vec{v})\rho_p]=-\epsilon_{ikp}\partial_j(\phi_k\rho_p)=-\epsilon_{ikp}(\partial_j\phi_k)\rho_p+W_{ij}(\vec{\phi}). \tag{A7}$$

To proceed, we must calculate $\partial_j\phi_k$. As Eq. (A6) shows, $\vec{\phi}$ has two components: the angular velocity of the center of mass of the particles that turn around the particle lying at $\vec{x}$ ($\vec{w}^c=\vec{\nabla}\times\vec{v}^c/2$) and the angular velocity of the particle at $\vec{x}(\vec{w}^u=\vec{\nabla}\times\vec{u}/2)$. Nevertheless, by hypotheses, at the edges of the grain, these velocities need not agree and a dissipative contribution may appear. Therefore, $\partial_j\phi_k$ is composed by two terms; one that takes care of the radial variations in $\vec{w}^c$ and $\vec{w}^u$, that we will note by $\partial_j^r\phi_k$, and another one that takes care of this dissipative change in angular velocities at the edges of the grain, which we will call $\partial_j^d\phi_k$. To calculate how $\partial_j^d\phi_k$ contributes to the derivative $\partial_j\phi_k$, we will use the radial coordinates, $\rho=(\Sigma_i(\rho_i)^2)^{1/2}$, where ρ_i has been defined below Eq. (16). Of course, the grain edge (at $\rho=\bar{\rho}$) divides the neighborhoods of the point $\vec{x}$ in two regions, one of them is the region occupied by the rotating grain, and another is the volume external to this region. So, it can be assumed that at $\bar{\rho}$, the discontinuity in $\vec{\phi}$ has the intensity $\Delta\vec{w}=\vec{w}^c-\vec{w}^u$, and it is described by a Heaviside function $H(\rho-\bar{\rho})$. Therefore,

$$\partial_j^d\phi_r\to\Delta\vec{w}\frac{dH(\rho-\bar{\rho})}{d\rho}\partial_j(\rho). \tag{A8}$$

But, from the definition of ρ, we have that

$$\partial_j(\rho)=\frac{\rho_i}{\rho}.$$

Furthermore, if we define the components of a unit vector $\vec{n}$ as,

$$n_i=\frac{\rho_i}{\rho}, \tag{A9}$$

we see that $\vec{n}$ points to the direction of the anisotropy of the grain, which agrees with the definition of a microscopic director, therefore,

$$\partial_j^d\phi_r=(\Omega_r^c-w_r^u)\,\delta(\rho-\bar{\rho})n_j,$$

giving,

$$\begin{aligned}\epsilon_{ikr}\langle(\partial_j^d\phi_r)\rho_k\rangle&=\epsilon_{ikr}\langle\rho_k((\Omega_r^c-w_r^u)\,\delta(\rho-\bar{\rho})n_j)\rangle\\&=-(W_{ik}(\vec{v}^c)\bar{\rho}_k+\dot{\bar{\rho}}_i)n_j,\end{aligned} \tag{A10}$$

where $\langle x\rangle$ is the statistical average of the random variable x. From Eq. (A9), we have that $\bar{\rho}_i=\bar{\rho}n_i$e $\dot{\bar{\rho}}_i=\rho\dot{n}_i$, therefore,

$$\epsilon_{ikr}\langle(\partial_j^d\phi_r)\rho_k\rangle=\bar{\rho}[W_{ik}(\vec{v}^c)n_k+\dot{n}_i]n_j,$$

giving,

$$\begin{aligned}\langle\partial_j v_i\rangle&=A_{ij}(\vec{v}^c)-\partial_j[W_{ip}(\vec{v})\rho_p]+[\partial_j W_{ip}(\vec{v}^c)]\rho_p\\&=A_{ij}(\vec{v}^c)+\bar{\rho}[W_{ik}(\vec{v}^c)n_k+\dot{n}_i]n_j+\epsilon_{ikp}(\partial_j^r\phi_k)\rho_p\\&\quad+[\partial_j W_{ip}(\vec{v}^c)]\rho_p.\end{aligned}$$

As the tree last terms are composed by higher derivatives of $\vec{v}^c$ and $\vec{w}^u$, we can write,

$$\langle\partial_j v_i\rangle=A_{ij}(\vec{v}^c)+\bar{\rho}[W_{ik}(\vec{v}^c)n_k+\dot{n}_i]n_j. \tag{A11}$$

This is the equation from which we will begin our calculation of the nematic viscosity.

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